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Simultaneous analysis, design and optimization of structures using the force method and supervised charged system search algorithm

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Abstract In this paper, the Charged System Search (CSS) algorithm and the force method are used for the simultaneous analysis and design of structures. Supervisor agents are introduced for the optimization procedure to enhance the exploration ability of the CSS. The presented method is applied to the design and analysis of some planer and space structures. A new formulation is presented for the objective function, and the accuracy and efficiency of the presented approach is examined by comparing the resulting design parameters and structural weight with those of other methods from literature.

1. Introduction

Developing methods with higher computation efficiency is a crucial subject in advanced engineering problems of a multi-physics nature. For instance, analyzing structures with larger numbers of members requires a larger memory size and longer computation time. In addition, this costly computation has to be repeated many times (typically over 10,000 times) because the cross-section size of the members is not determined in the early stages of designing such structures. Therefore, reducing the size of structural matrices and eliminating undue repetitions in the design and analysis procedures can lead to a considerable reduction in computation efficiency [1,2]. In this paper, this goal is achieved utilizing meta-heuristics algorithms which minimize the energy function indirectly. Besides, the design procedure and minimizing the weight of the structure is added to the analysis procedure. One of the most reliable meta-heuristic methods recently developed is the Charged System Search (CSS) [3,4], which we use here. In this paper, supervisor agents are considered to increase the exploration ability of the CSS algorithm. This method is called supervised CSS, abbreviated as SCSS. Also, a new formulation of the penalty function is made to improve the performance of the supervised CSS.

Designing structures with minimum weight can be achieved by using minimum energy methods and members with pre-defined stress ratios [5], instead of the direct solution of classic equations. This results in avoiding not only repetitive computations in the design and analysis, but, also, avoiding the computation of the inverse of the large matrices. To this end, one needs to formulate the equations based on the minimum energy principle, and employ them in an efficient optimization algorithm. Combining the SCSS algorithm and the force method provides a suitable means for this purpose, since the former provides the optimization algorithm and the latter can be used to derive the energy equations.

In the first part of this paper, supervisor agents are introduced. In the second part, energy formulation based on the force method is derived and the supervised SCSS algorithm is applied to the analysis procedure. In the third part, using the SCSS and prescribed stress ratios, structures are analyzed and designed, and finally, in the last part, weight minimization is performed by imposing the analysis procedure as a constraint onto the SCSS. In recent years, the CSS has been applied successfully in many engineering optimization problems. In this method, CSS has performed very well and improved all resulted

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design parameters and weights achieved by other algorithms. Large-scale structures are analyzed and designed in this paper in order to show the accuracy of the method when applied to different kinds of structures.

2. Supervised CSS algorithm

In the CSS algorithm, each vector of the variables is an agent that moves through the search space and finds the minimal solutions [3,4]. Throughout the search process, an agent might go to a coordinate in the search space that already has been searched by the same agent or another. If this coordinate has a good fitness, it will be saved in the charged memory [3], but if this coordinate does not have a good fitness, it will not be saved anywhere. Therefore, this step of the search process becomes redundant. This unnecessary step adversely affects the exploration ability of the algorithm. In this paper, the supervisor agents are introduced to improve the exploration ability of the CSS algorithm. The supervisor agent is an independent agent of constant value that repels the agent if its coordinate has a bad fitness, or attracts the agents if its coordinate has a good fitness. This procedure is repeated in all of the iterations and gives an overall view of the search space. The number of supervisor agents is selected at the beginning of the algorithm, and then their constant coordinates in the search space are determined as follows:

$$xs_{j,i} = \frac{(i-1)[x_{\max,j} - x_{\min,j}]}{NOSA - 1} + x_{\min,j}, \tag{1}$$

where $NOSA$ is the number of supervisor agents, $xs_{j,i}$ is the jth variable of the ith supervisor agent, and $x_{\min,j}$ and $x_{\max,j}$ are the minimum and maximum limits of the jth variable. The kind of force for these agents is determined as:

$$p = \log\left(\frac{\overline{\text{fit}}}{\text{fit}_i}\right), \tag{2}$$

where p is the same as the parameter in the original version of the CSS [3], fit_i is equal to the fitness value of the ith supervisor agent and $\overline{\text{fit}}$ is the average value of the fitness of the normal agents. Calculating other properties of the supervisor agents, such as force and radius, are similar to the standard CSS algorithm [3]. Supervisor agents do not move from their coordinate determined from Eq. (1), yet they apply additional forces on the normal agents. By doing so, they determine the fitness values of their fixed coordinate and its neighborhood, resulting in a better exploration ability of the CSS algorithm.

3. Analysis by force method and charged system search

In the presented approach, the force method is applied to analyze structures. Since this method leads to less numbers of unknowns, it is preferred to the displacement method. In the force method, the redundant forces are unknowns, whereas in the displacement method, the nodal displacements are unknowns. In this method [1,2,5], the energy relationships of the structure that satisfy compatibility, force–displacement and equilibrium conditions are derived, and then, minimized, using the SCSS. Suppose $\{\boldsymbol{p}\} = \{p_1, p_2, \dots, p_n\}^t$ is the vector of nodal forces, $\{\boldsymbol{q}\} = \{q_1, q_2, \dots, q_n\}^t$ is the vector of redundant forces, and $\{\boldsymbol{r}\} = \{s_1, s_2, \dots, s_m\}^t$ comprises the internal forces of the members. The equilibrium condition results in the following equation [1,2]:

$$\boldsymbol{r} = \boldsymbol{B_0 p} + \boldsymbol{B_1 q} = \begin{bmatrix}\boldsymbol{B_0} & \boldsymbol{B_1}\end{bmatrix}\begin{bmatrix}\boldsymbol{p}\\ \boldsymbol{q}\end{bmatrix}, \tag{3}$$

where $\boldsymbol{B}_0$ and $\boldsymbol{B}_1$ are rectangular matrices, each having m rows, and n and DSI columns, respectively. m, n and DSI are the number of members, number of nodes and degree of statical indeterminacy of the structures, respectively. In addition, the complementary energy function is:

$$U^c = \frac{1}{2}\boldsymbol{r^t}\boldsymbol{F}_m\boldsymbol{r}, \tag{4}$$

where $[\boldsymbol{F}_m]$ is the unassembled flexibility matrix of the structure. This matrix relates the member internal forces to the nodal displacement, which can be obtained using Hooke's law, as discussed in [1]. According to the Castigliano principle, a group of redundant forces that minimize the complementary energy function is the exact solution that satisfies the compatibility condition. By substituting $\{\boldsymbol{r}\}$ from Eq. (3) in Eq. (4), the following equation is obtained:

$$U^c = \frac{1}{2}\begin{bmatrix}\boldsymbol{p^t} & \boldsymbol{q^t}\end{bmatrix}\boldsymbol{H}\begin{bmatrix}\boldsymbol{p}\\ \boldsymbol{q}\end{bmatrix}, \tag{5}$$

where $\boldsymbol{H} = \begin{bmatrix}\boldsymbol{B_0} & \boldsymbol{B_0}\end{bmatrix}^{\boldsymbol{t}}\boldsymbol{F_m}\begin{bmatrix}\boldsymbol{B_0} & \boldsymbol{B_1}\end{bmatrix}$. Decomposing matrix $[\boldsymbol{H}]$ into four submatrices leads to:

$$U^c = \frac{1}{2}(\{\boldsymbol{p}\}^{\boldsymbol{t}}[\boldsymbol{H_{pp}}]\{\boldsymbol{p}\} + \{\boldsymbol{p}\}^{\boldsymbol{t}}[\boldsymbol{H_{pq}}]\{\boldsymbol{q}\} + \{\boldsymbol{q}\}^{\boldsymbol{t}}[\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + \{\boldsymbol{q}\}^{\boldsymbol{t}}[\boldsymbol{H_{qq}}]\{\boldsymbol{q}\}). \tag{6}$$

As mentioned above, $\{\boldsymbol{p}\}$ is the nodal force and $\{\boldsymbol{q}\}$ is the vector of the redundant forces. In the classical method, the derivative of U^c, in terms of $\{\boldsymbol{q}\}$, is calculated and is equaled to zero, in order to find a set of redundant forces that minimizes the complementary energy value leading to:

$$\begin{bmatrix}\boldsymbol{H_{qq}}\end{bmatrix}\{\boldsymbol{q}\} + \begin{bmatrix}\boldsymbol{H_{qp}}\end{bmatrix}\{\boldsymbol{p}\} = 0, \tag{7a}$$

$$\{\boldsymbol{q}\} = -[\boldsymbol{H_{qq}}]^{-\boldsymbol{1}}[\boldsymbol{H_{qp}}]\{\boldsymbol{p}\}. \tag{7b}$$

Since $[\boldsymbol{H}]$ is symmetric, $[\boldsymbol{H_{qp}}]^t = [\boldsymbol{H_{pq}}]$ [5].

Accordingly, in the classical method, the inverse of $[\boldsymbol{H_{qq}}]$ needs to be calculated. This is a difficult task, and requires extensive computer memory, especially in the case of large-scale structures. Therefore, finding $\{\boldsymbol{q}\}$ that minimizes the complementary energy without calculating the inverse of $[\boldsymbol{H_{qp}}]$ reduces the computation time and computer memory. The first term of Eq. (6) is constant and the second and third terms are equal. It can be shown that the third and fourth terms of U^c are symmetric. Therefore:

$$F_u = \{\boldsymbol{q}\}^{\boldsymbol{t}}\begin{bmatrix}\boldsymbol{H_{qp}}\end{bmatrix}\{\boldsymbol{p}\}, \tag{8}$$

is the equation that should be minimized [5].

An enhanced charged system search [4] is used to minimize Eq. (8). In this part, the force method analysis is applied to different types of structures to illustrate the performance of the method.

Case study 1.

The first example is an eleven-member truss with three degrees of statical indeterminacy, as shown in Figure 1. Consequently, the energy function includes three variables.

The classical method that calculates the exact and minimum amount of U^c leads to 419.8475, whereas, using the present approach with CSS, $U^c = 419.8476$ is obtained and $\{\boldsymbol{q}\}$ is calculated as:

$$\{\boldsymbol{q}\} = \{4.6394\ \ -3.7629\ 8.1900\}^t.$$

The optimization history is shown in Figure 2. The number of agents is selected as 20.

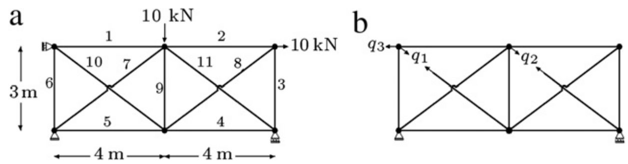


Figure 1: A simple truss and the selected basic structure (Case study 1): (a) A planar truss; (b) The selected basic structure.

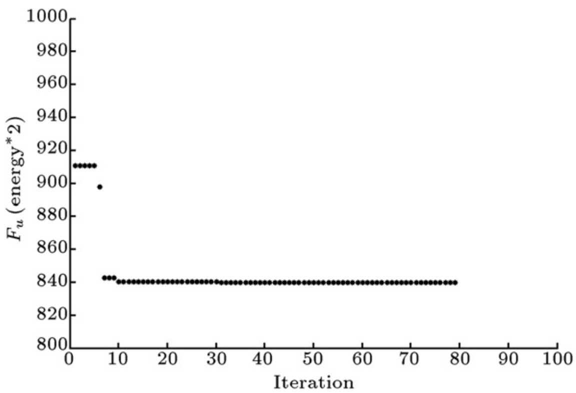


Figure 2: Variation of F_U versus the number of iterations in the eleven-member truss (Case study 1).

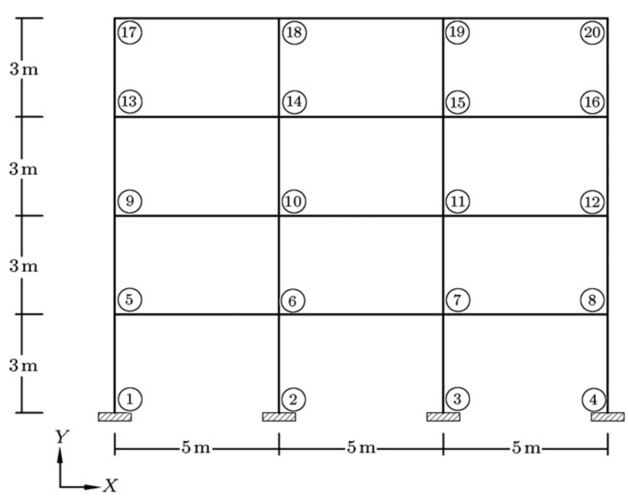


Figure 3: An unbraced planar frame (Case study 2).

Case study 2.

The second example is an unbraced planar frame with constant EI having 36 degrees of statical indeterminacy, as shown in Figure 3. In this example, the axial force, shear and moment in the first node of the beams are considered as the redundant forces. As a result, the energy function includes 36 variables. Note that only the bending energy is considered as the energy of the frame. The loading condition is considered as:

1. A load 10 kN in the y direction at nodes 8–11;
2. A load 10 kN in the x direction at nodes 8–11;
3. A bending moment 10 kN m in the $x - y$ surface at nodes 8–11.

The exact calculation of U^c leads to 1234.8, while it is $U^c = 1249.2$ utilizing the CSS algorithm. Figure 4 shows the variation of F_U versus the number of iterations. As shown above, there is very close agreement between the exact and calculated values for the energy function, verifying the accuracy of the algorithm. In this case, the redundant forces are obtained as follows:

$$\{\boldsymbol{q}\} = \{1.1275, 5.3155, 14.0096, 2.4854, 4.8316, 12.0549, 4.0405, 4.2845, 10.7913, -3.0551, 1.2459, 2.9740, -4.0016, 1.3874, 3.2303, 5.5762, 1.4122, 1.3221, 0.0660, 0.2315, 0.4707, 0.1680, 0.2155, 0.4678, 0.4265, 0.1987, 0.2503, -0.1444, 0.0425, -0.0728, 0.0540, 0.0052, 0.0351, 0.0373, 0.0847, 0.0901\}^t.$$

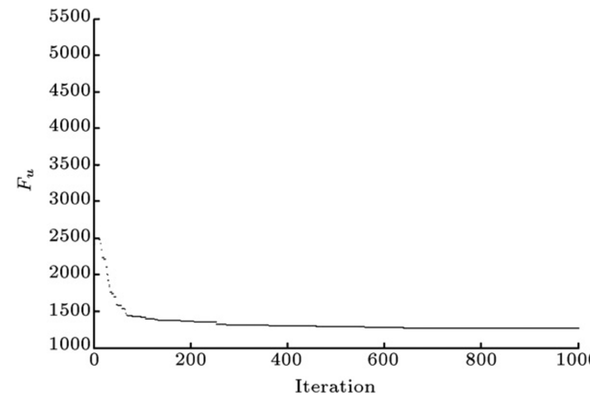


Figure 4: Variation of $\boldsymbol{F}_U$ versus the number of iterations in the unbraced planar frame analysis (Case study 2).

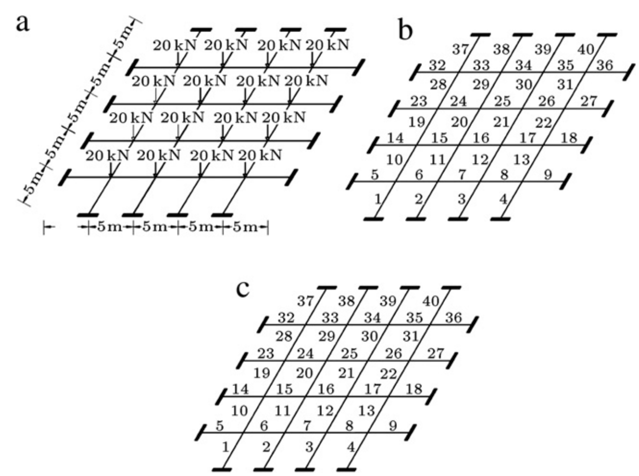


Figure 5: A 40-element grillage (Case study 3): (a) Geometry; (b) Node and element ordering; (c) Basic structure.

Case study 3.

In the third example, a 40-element grilling system is considered to illustrate the accuracy of the force method and CSS in analyzing space frames. Geometry, nodal loads and basic structure are shown in Figure 5. Torsion and shear in the z direction, and moment around the axis with a greater moment of inertia in each member are considered redundant forces.

Both the torsion and bending energies are considered energy functions in this structure. G, I and E are constant for members and Poisson's ratio (υ) is considered 0.3. The cross-sections of members are considered to be W-sections, as given in LRFD-AISC. Using least square regression, the polar moment of inertia (J) is expressed as a function of the moment of inertia (I) for the mentioned sections:

$$J = 1.04I. \quad (9)$$

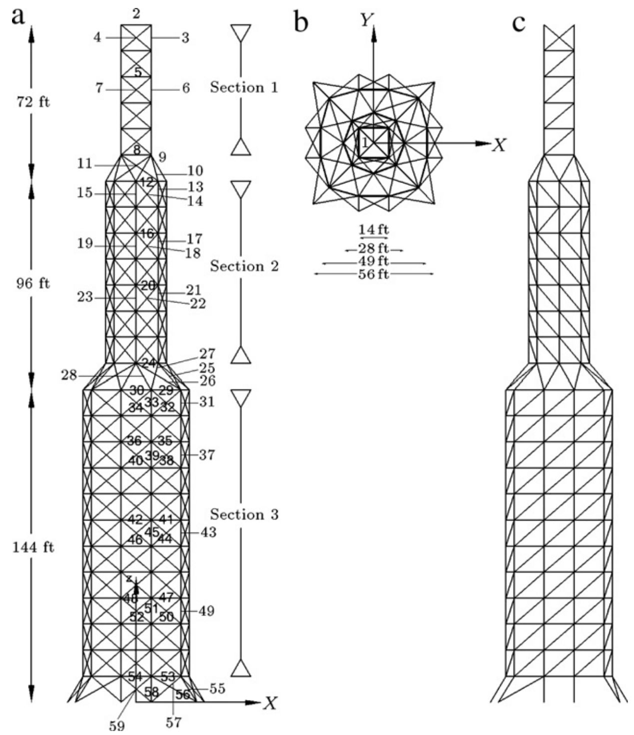


Figure 6: A 26-story tower: (a) Geometry and grouping; (b) Top view; (c) Basic structure (Case studies 4 and 11).

Also:

$$E = 2G(1 + \upsilon). \tag{10}$$

By substituting Eqs. (9) and (10) in $[\boldsymbol{F}_m]$, the energy function is derived. The exact calculation of energy using the classical method leads to 170 840, whereas, using the present approach, $U^c = 177\,460$ is obtained. The redundant forces, $\{\boldsymbol{q}\}$, are shown in Table 1.

Case study 4.

The last example of this part is a 26-story tower with 246 degrees of statical indeterminacy selected from [6]; as shown in Figure 6(a) and (b), the energy function has 246 unknowns. The cross section and module of elasticity for all the elements are considered constant and equal. Geometry and basic structure are shown in Figure 6(c).

The loading on the structure consists of:

1. The vertical load at each node in the first section is equal to −3 kips (−13.344 kN).
2. The vertical load at each node in the second section is equal to −6 kips (−26.688 kN).
3. The vertical load at each node in the third section is equal to −9 kips (−40.032 kN).
4. The horizontal load at each node on the right side in the x direction is equal to −1 kips (−4.448 kN).
5. The horizontal load at each node on the left side in the x direction is equal to 1.5 kips (6.672 kN).
6. The horizontal load at each node on the front side in the y direction is equal to −1 kips (−4.448 kN).
7. The horizontal load at each node on the back side in the y direction is equal to 1 kips (4.448 kN).

In this example, the exact calculation of the energy function leads to 1.8008e7, and it is obtained as 1.8252e7 using the force method and CSS, which is very close to the exact value.

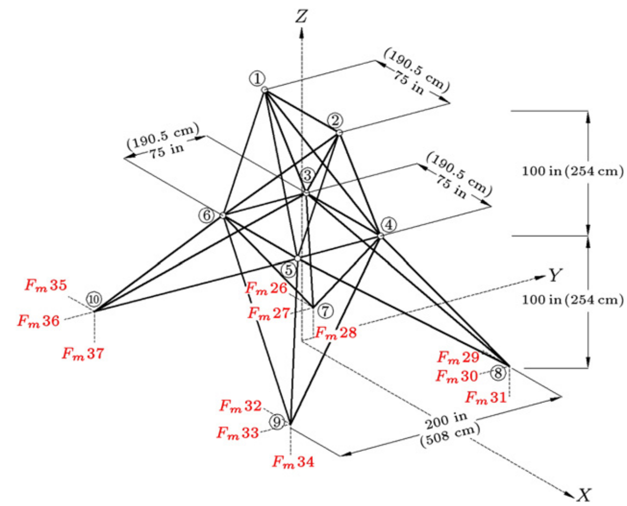


Figure 7: A twenty five-bar space truss with elastic boundary condition (Case study 5).

Case study 5.

The elastic boundary condition is a common condition in finite element modeling. In modeling this condition using the force method, each support force can be substituted by a rod in the direction of the corresponding support force. In the case of the elastic boundary condition, this rod should have the same flexibility or stiffness magnitude as that of the elastic support. In the case of rigid constraint, the substituted rods should have zero flexibility. In the force method, these substituted rods will add new columns to the end of the equilibrium matrices [1] and internal forces for these rods indicate the reaction forces of the supports. Besides, these rod flexibilities, corresponding to the support flexibilities, will be added to the end of the flexibility matrix of the structure, and other concepts and formulation of the structure will be identical to previous methods. Then, the energy formulation will be derived, similar to the previous case, and will be minimized using the CSS algorithm. One example of this approach using the CSS algorithm is provided in the following.

In this example, a twenty five-bar space truss with elastic supports is considered, as shown in Figure 7. The constraints are substituted by imaginary rods. The flexibility of the support in node number 10 is considered 4.4483^{-1} kN^{-1}, and other constraints are considered rigid. The loading condition is considered as:

1. 80, 120 and 30 kN in node number 1 in the direction of the X, Y, and Z axes, respectively.
2. 60, 100 and 30 kN in node number 2 in the direction of the X, Y, and Z axes, respectively.
3. 30 kN in node number 3 in the direction of the X axis.
4. 30 kN in node number 6 in the direction of the X axis.

Redundant forces are considered as the internal forces of the 2nd, 23rd, 25th, 22nd, 24th, 5th and 21st members. The redundant force vector is calculated by the CSS as follows:

$$\{\boldsymbol{q}\} = \{-144350\ -300\ 61850\ 263110$$
$$-254520\ 114490\ 86370\}N.$$

The exact calculation of energy using the classical method leads to 6117.0, whereas, using the present approach, $U^c = 6117.5$ is obtained.

Table 1: The calculated redundant forces of 40-element grilling system (Case study 3) $\times\ 10^4$ (N).

$\boldsymbol{q}_1$	−0.914	$\boldsymbol{q}_{13}$	−0.7939	$\boldsymbol{q}_{25}$	−0.0497	$\boldsymbol{q}_{37}$	−0.0312	$\boldsymbol{q}_{49}$	1.2002	$\boldsymbol{q}_{61}$	0.1051
$\boldsymbol{q}_2$	0.2167	$\boldsymbol{q}_{14}$	0.2277	$\boldsymbol{q}_{26}$	0.0678	$\boldsymbol{q}_{38}$	−5.1336	$\boldsymbol{q}_{50}$	−5.6626	$\boldsymbol{q}_{62}$	−3.0445
$\boldsymbol{q}_3$	−3.9005	$\boldsymbol{q}_{15}$	3.3177	$\boldsymbol{q}_{27}$	5.0685	$\boldsymbol{q}_{39}$	−0.0287	$\boldsymbol{q}_{51}$	0.1194	$\boldsymbol{q}_{63}$	1.9832
$\boldsymbol{q}_4$	−0.6323	$\boldsymbol{q}_{16}$	0.1725	$\boldsymbol{q}_{28}$	1.0572	$\boldsymbol{q}_{40}$	0.5316	$\boldsymbol{q}_{52}$	1.1286	$\boldsymbol{q}_{64}$	−0.0718
$\boldsymbol{q}_5$	0.314	$\boldsymbol{q}_{17}$	−1.4645	$\boldsymbol{q}_{29}$	−0.1714	$\boldsymbol{q}_{41}$	−1.9493	$\boldsymbol{q}_{53}$	−5.547	$\boldsymbol{q}_{65}$	0.2401
$\boldsymbol{q}_6$	−0.3381	$\boldsymbol{q}_{18}$	−0.8168	$\boldsymbol{q}_{30}$	5.5207	$\boldsymbol{q}_{42}$	0.0136	$\boldsymbol{q}_{54}$	−0.17	$\boldsymbol{q}_{66}$	1.3579
$\boldsymbol{q}_7$	0.1307	$\boldsymbol{q}_{19}$	−0.0084	$\boldsymbol{q}_{31}$	0.4753	$\boldsymbol{q}_{43}$	−0.0397	$\boldsymbol{q}_{55}$	0.7119	$\boldsymbol{q}_{67}$	0.0941
$\boldsymbol{q}_8$	−0.0469	$\boldsymbol{q}_{20}$	−1.8335	$\boldsymbol{q}_{32}$	−4.0345	$\boldsymbol{q}_{44}$	0.0061	$\boldsymbol{q}_{56}$	−4.0377	$\boldsymbol{q}_{68}$	−2.4965
$\boldsymbol{q}_9$	2.8322	$\boldsymbol{q}_{21}$	0.7346	$\boldsymbol{q}_{33}$	0.0442	$\boldsymbol{q}_{45}$	4.5725	$\boldsymbol{q}_{57}$	−0.2541	$\boldsymbol{q}_{69}$	−0.2361
$\boldsymbol{q}_{10}$	0.4806	$\boldsymbol{q}_{22}$	−1.0314	$\boldsymbol{q}_{34}$	−0.3564	$\boldsymbol{q}_{46}$	−0.2432	$\boldsymbol{q}_{58}$	0.0398	$\boldsymbol{q}_{70}$	−0.8848
$\boldsymbol{q}_{11}$	−0.3335	$\boldsymbol{q}_{23}$	3.6083	$\boldsymbol{q}_{35}$	−3.7443	$\boldsymbol{q}_{47}$	−1.6436	$\boldsymbol{q}_{59}$	−6.1707	$\boldsymbol{q}_{71}$	−3.9475
$\boldsymbol{q}_{12}$	2.1219	$\boldsymbol{q}_{24}$	0.0769	$\boldsymbol{q}_{36}$	0.055	$\boldsymbol{q}_{48}$	0.296	$\boldsymbol{q}_{60}$	2.1362	$\boldsymbol{q}_{72}$	0.2642

Table 2: Design data for the 11-bar planar truss (Case study 6).

Design variables
Redundant and size variables q_1; q_2; q_3; A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10}; A_{11}
Material and section property
Young's modulus is assumed to be constant. Density of the material: $\rho = 0.00277$ kg/cm^3 $= 0.1$ lb/in^3 $A = 0.4h^2$, $r = \sqrt{0.4A}$, thickness $t = 0.1h$.
Constraint data Stress ratios
Case 1 $C = \{0.9, 0.8, 0.85, 0.8, 0.9, 0.85, 0.95, 0.9, 0.8, 0.9, 0.95\}$ Case 2 $c_i = 1$; $i = 1, \ldots, 11$
For tensile members
$F_a \leq 0.6\, F_y$ and $\lambda_i \leq 300$
For compressive members
$\lambda_i \leq 200$ $F_a = \dfrac{\left[\left(1 - \frac{\lambda_i^2}{2C_c^2}\right)F_y\right]}{\left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3}\right)}$ for $\lambda_i \leq C_c$ $F_a = \frac{12\pi^2 E}{23\lambda_i^2}$ for $\lambda_i \leq C_c$
Stress constraints
$\sigma_i < 234.43$ MPa; $i = 1, \ldots, 11$

4. Procedure of structural design using force method and the CSS

In this section, design and optimization procedures are added to the analysis presented in the previous section. There are two major approaches to formulate the objective function in the simultaneous analysis and design of an optimal structure:

1. Using the pre-selected stress ratio.
2. Minimizing the structure weight.

4.1. Pre-selected stress ratio

In this approach [5], a pre-selected stress ratio is assumed for each member, and then, the complementary energy is minimized as the objective function. If the cross sections, A_i ($i = 1, \ldots, m$), are known, then the analysis can be performed using a meta-heuristics method, such as CSS, described in Section 3.

However, usually, the cross-sectional areas are not determined at the beginning of the design procedure. This problem leads to a new formulation of the complementary energy that eliminates A_i ($i = 1, \ldots, m$) from the energy function [5].

Each agent in the CSS is a vector of redundant forces. Moreover, according to Eq. (3), the internal forces of members, $\{\boldsymbol{r}\}$, are obtained from the selected agents. The ratio between the stress in each member (σ_i) and its corresponding allowable stress (σ_a) is defined as C:

$$C = \frac{\sigma_i}{\sigma_a}, \tag{11}$$

where $\sigma_i = \frac{r_i}{A_i}$. By substituting σ_i in Eq. (11), the cross-section area of each member is obtained in terms of the internal force, r_i, stress ratio, C, and the allowable stress, σ_a, as:

$$A_i = \frac{r_i}{C\sigma_a}. \tag{12}$$

Consequently, one can express the unassembled flexibility matrix of each member as a function of L, which is equal to the length of the members. E is the modulus of elasticity, $\boldsymbol{q}$ and C, as follows:

$$\boldsymbol{F_m} = \frac{L}{EA} = \frac{1}{E\boldsymbol{f}(\boldsymbol{r}, L, C)} = \boldsymbol{g}(\boldsymbol{q}, C, L, E). \tag{13}$$

Substituting $\boldsymbol{F}_m$ in Eq. (4) leads to the elimination of A_i from the formulation of the complimentary energy:

$$\begin{aligned} \mathrm{Min} U^c = {} & \frac{1}{2E}\begin{bmatrix}\boldsymbol{p} & \boldsymbol{q}\end{bmatrix}^{\boldsymbol{t}}\begin{bmatrix}\boldsymbol{B_0} & \boldsymbol{B_1}\end{bmatrix}^{\boldsymbol{t}}[\boldsymbol{g}(\boldsymbol{q}, C, L)] \\ & \times \begin{bmatrix}\boldsymbol{B_0} & \boldsymbol{B_1}\end{bmatrix}\begin{bmatrix}\boldsymbol{p} & \boldsymbol{q}\end{bmatrix}. \end{aligned} \tag{14}$$

Pre-selected stress ratios are a parameter controlling the weight of the structure and stress constraint, simultaneously. Therefore, by minimizing the energy function in the analysis procedure, weight optimization and stress constraint satisfaction are fulfilled.

Case study 6.

As an example, consider the truss shown in Figure 8. This truss is designed with the constraints explained in Table 2 and using Eq. (14) as the objective function. In this example, two cases are considered. In Case I, the stress ratios of the members are different, whereas, in Case II, it is assumed to be constant for all members. For the sake of simplicity, the cross-sections are selected as hollow squares, as shown in Figure 9. In this example, a population of 20 agents is considered in the CSS algorithm. The magnitude of A_i is determined considering the selected values of C_i. Enhanced CSS with a supervisor agent is utilized in the simultaneous analysis and design of this structure and the results are shown in Tables 3 and 4. The convergence history is shown in Figure 10. To verify the efficiency of the present method, and by combining the CSS algorithm and force method in minimizing the structural weight, the design parameters and redundant forces obtained from CSS, are compared to those computed using the Genetic Algorithm (GA), reported by Kaveh and Rahami in [5]. The

Table 3: Optimal design comparison for the 11-bar truss (Case study 6; Case I).

Weight (N)	Redundant variables $\times 10^3$ (N)			Size variable (cm^2)										
	q_1	q_2	q_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
2136.25	123.04	−5.04	244.69	11.55	13.36	41.20	4.44	4.44	42.51	6.94	9.15	61.02	9.71	17.51

Table 4: Optimal design comparison for the 11-bar truss (Case study 6; Case II).

Weight (N)	Redundant variables $\times 10^3$ (N)			Size variable (cm^2)										
	q_1	q_2	q_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
1914.84	94.04	−5.41e−5	198.66	11.55	13.36	41.20	4.44	4.44	42.51	6.94	9.15	61.02	9.71	17.51

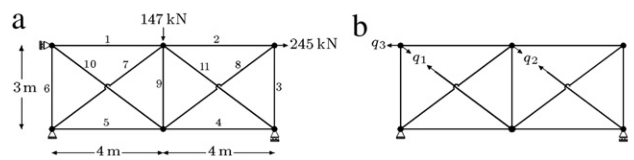


Figure 8: A simple truss with pre-selected stress ratios (Case study 6): (a) Geometry; (b) Basic structure.

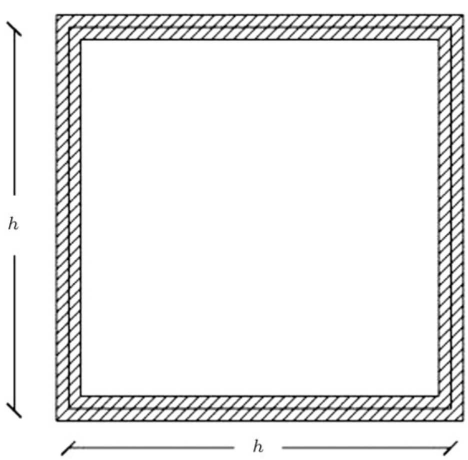


Figure 9: A hollow square cross-section (Case study 6).

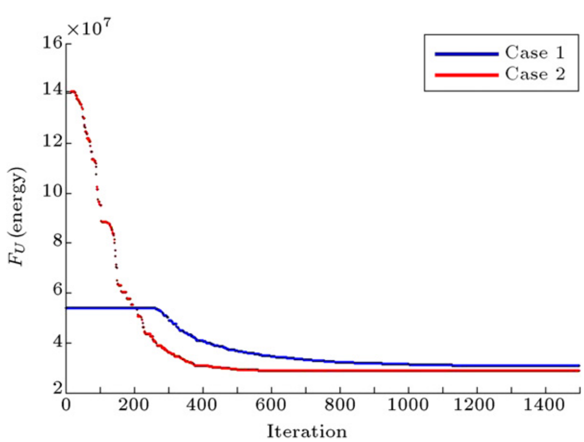


Figure 10: Variation of F_U versus the iteration in the design procedure for the 11-member truss (Case study 6).

comparison results are shown in Tables 3 and 4 for Case I and Case II, respectively.

In this example, the exact calculation of the energy function leads to 6.5989e5, and it is obtained as 6.6056e5 using the force method and CSS for Case I. Besides, the exact calculation of the energy function leads to 7.5368140e5, and it is obtained as 7.5368147e5 using the force method and CSS for Case II. The close agreement between these values verifies the accuracy of the calculated redundant forces shown in Tables 3 and 4 for Cases I and II, respectively. Also, variation of F_U versus the iteration is shown in Figure 10.

4.1.1. Fully Stress Design (FSD) for statically indeterminate structures

In this part, the presented CSS and force method is applied to an Optimality Criteria Method (OCM) [7], namely, Fully Stressed Design (FSD). FSD leads to a correct optimal weight for statically determinate structures under a single load condition. In the FSD, all members are supposed to be subjected to their maximal allowable stresses [5]. Achieving such a design for an indeterminate structure with fixed geometry is not always possible. Even by changing the geometry, a FSD may not be achieved. Here, a formulation presented by Kaveh and Rahami [5] is used for indirect analysis in the process of optimization. This formulation can be applied to all types of structure, however, a truss with the following strain energy is considered:

$$U^c = \sum \frac{P^2 L}{EA} = \sum \frac{\gamma P^2 LA}{\gamma EA^2} = \frac{1}{\gamma E} \sum \sigma_i^2 w_i. \tag{15}$$

It should be noted that for constant E and γ, defined as the module of elasticity and mass density of the material, respectively, the minimum weight can be achieved only when the stresses in all the members are identical. In Eq. (15), P is the member axial force, A is the cross-sectional area of the member, L equals the member length, σ is the member stress and w is the member weight. Therefore, in Eq. (15), the term corresponding with the stresses, i.e. σ_i^2, may be moved out of the summation. On the other hand, in the design procedure, one can consider the fully stress constraint instead of minimum weight. This is because the minimum weight corresponds to a structure whose members are all subjected to their maximum allowable stress.

Case study 7.

As an example, consider the structure shown in Figure 11, selected from [7]. The design and member size constraints are reported in Table 5. Redundant forces in this example are selected as internal forces in members 1 and 9. Twenty agents are selected in the CSS algorithm.

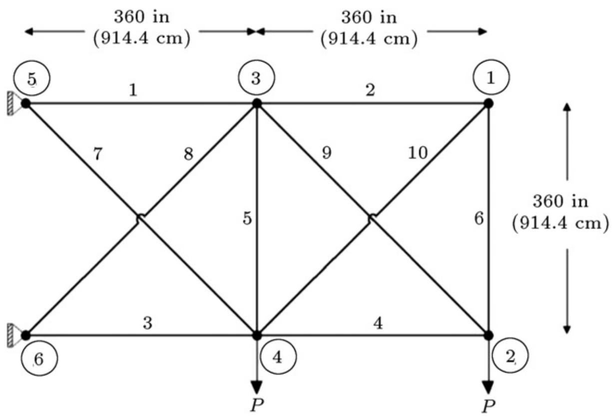


Figure 11: A ten-bar truss example (Case studies 7 and 8 [7]).

Table 5: Design data for the 10-bar planar truss (Case study 7).

Loading			
Node	Px: kips (kN)	Py: kips (kN)	Pz: kips (kN)
2	0	−100 (−444.8)	0
4	0	−100 (−444.8)	0
Design variables			
Variables: q_1; q_2 (and A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10} in Case 3)			
Material property and constraint data			
Young's modulus: $E = 1e7$ psi $= 6.895e7$ MPa. Density of the material: $\rho = 0.1$ lb/in$^3 = 0.00277$ kg/cm^3 For all members: $A_i \geq 0.1$ in^2; $i = 1, \ldots, 10$			
Stress constraints			
(a) FSD Case 1: $\|\sigma_i\| \leq 25$ ksi (172.375 MPa); $i = 1, \ldots, 10$ Case 2: $\|\sigma_i\| \leq 25$ ksi; $i = 1, \ldots, 8, 10$ and $\|\sigma_9\| \leq 50$ ksi (344.75 MPa) (b) Weight minimization Case 3: $\|\sigma_i\| \leq 25$ ksi; $i = 1, \ldots, 8, 10$ and $\|\sigma_9\| \leq 50$ ksi (344.75 MPa)			

Table 6: Results of the 10-bar planar truss (Case study 7; Cases 1–3).

Case 1 (FSD)
$A = \{7.94\ 0.10\ 8.05\ 3.91\ 0.10\ 0.10\ 5.73\ 5.57\ 5.54\ 0.11\}$ in^2 $W = 1591.8$ lb
Case 2 (FSD)
$A = \{4.13\ 3.88\ 11.86\ 0.11\ 0.10\ 3.88\ 11.12\ 0.18\ 0.10\ 5.49\}$ in^2 $W = 1724.6$ lb
Case 3 (weight minimization)
$A = \{7.77\ 0.24\ 8.25\ 3.79\ 0.1011\ 0.22\ 5.97\ 5.41\ 3.67\ 0.31\}$ in^2 $W = 1516.2$ lb

Table 7: Optimal design comparision for the 10-bar truss (Case study 7).

Method	Kaveh and Rahami (GA) [5]	Kaveh and Hassani (ACO) [8]	Present work
Best weight (Case 1) lb (kN)	1593.5 (7.087)	1593.5 (7.087)	1591.8 (7.080)
Best weight (Case 2) lb (kN)	1723.5 (7.666)	1723.5 (7.666)	1724.6 (7.671)
Best weight (Case 3) lb (kN)	1519.2 (6.757)	1519.2 (6.757)	1516.2 (6.744)

4.2. Minimum weight

In the second approach of simultaneous design and analysis of structures, the objective function is the weight of the structure, and the equilibrium, compatibility, and force/displacement conditions are the constraints. In summary, all these three conditions are called analysis criteria for simplicity. Other constraints, such as stress, displacement, dynamical properties, and etc. can also be imposed onto the fitness value. The penalty function is the most common approach to satisfy the constraints. The penalty function imposes a penalty onto the fitness value of the solution, if the constraint is not satisfied:

$$f = A + \alpha B. \tag{16}$$

In Eq. (16), f is the fitness value, A is the objective function and B is the penalty function; α is often selected as a big number. According to this equation, when B goes to zero and A goes to its minimum value, f goes to the minimum value of fitness. However, since the minimum complementary energy is not zero, this form of penalty function cannot be used. In this case, the weight of the structure is minimum, while the corresponding U^c is not minimum, i.e. the structure is not analyzed yet. Also, a small value of α does not guarantee the minimum value of B. On the other hand, in a structure that is in equilibrium and in a compatibility state, the sum of the complementary energy, U^c, and the strain energy, U, is zero. Therefore, instead of the complementary energy, the sum of the complementary energy and the strain energy is used as the analysis criteria and is imposed onto CSS as a constraint. The strain energy is a function of nodal displacements, as follows [5]:

$$\{\boldsymbol{d}\} = [\boldsymbol{B_0}]^{\boldsymbol{t}}\,[\boldsymbol{F_m}]\,([\boldsymbol{B_0}]\,\{\boldsymbol{p}\} + [\boldsymbol{B_1}]\,\{\boldsymbol{q}\}). \tag{17}$$

And;

$$U = \frac{1}{2}\,\{\boldsymbol{d}\}^{\boldsymbol{t}}\,[\boldsymbol{K}]\,\{\boldsymbol{d}\} - \{\boldsymbol{d}\}^{\boldsymbol{t}}\,\{\boldsymbol{F}\}\,, \tag{18}$$

where $[\boldsymbol{K}]$ is the stiffness matrix and $\{\boldsymbol{F}\}$ is the nodal force vector. For equilibrium, U is negative and $U + U^c$ is equal to zero. This formulation is used for the ten-bar truss example (Case study 7) of Case III. Table 6 shows the results. Twenty agents are selected in the CSS algorithm. Also, the resulting

Table 8: Design data for the ten-bar planar truss (Case study 8).

Material property and constraint data
Young's modulus: $E = 1e7$ psi $= 6.895e7$ MPa. Density of the material: $\rho = 0.1$ lb/in$^3 = 0.00277$ kg/cm^3
Stress constraints
$\|\sigma_i\| \leq 25$ ksi (172.375 MPa); $i = 1, \ldots, 10$
Nodal displacement constraint in all directions of the co-ordinate system
$\|\Delta_i\| \leq 2$ in (5.08 cm); $i = 1, \ldots, 4$
List of the available profiles *Case* 1: (Discrete sections)
$A_i = \{1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5\}$ in^2 $A_i = \{10.4516, 11.6129, 12.8387, 13.7419, 15.3548, 16.9032, 16.9677, 18.5806, 18.9032, 19.9354, 20.1935, 21.8064, 22.3871, 22.9032, 23.4193, 24.7741, 24.9677, 25.0322, 26.9677, 27.2258, 28.9677, 29.6128, 30.9677, 32.0645, 33.0322, 37.0322, 46.5806, 51.4193, 74.1934, 87.0966, 89.6772, 91.6127, 99.9998, 103.2256, 109.0320, 121.2901, 128.3868, 141.9352, 147.7416, 170.9674, 193.5480, 216.1286\}$ cm^2
Case 2: (Continuous sections)
$0.1 \leq A_i \leq 35$ in^2 (225.8960) cm^2; $i = 1, \ldots, 10$

Table 9: Optimal design comparison for the 10-bar planar truss (Case study 8) with discrete cross section.

Method	Weight lb (kN)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Kaveh and Rahami [5]	5490.738 (24.4228)	33.5	1.62	22.90	14.2	1.62	1.62	7.97	22.9	22.00	1.62
Shih [9]	5491.71 (24.4271)	33.50	1.62	22.90	15.50	1.62	1.62	7.97	22.00	22.00	1.62
Rajeev [10]	5613.84 (24.9704)	33.50	1.62	22.90	15.50	1.62	1.62	14.20	19.90	19.90	2.62
Kaveh and Hassani [8]	5517.72 (24.5702)	33.50	1.62	22.90	14.2	1.62	1.62	11.5	22.00	19.90	1.62
Present work	5475.40 (24.3817)	30.00	1.62	22.90	16.00	1.62	1.62	7.97	22.90	22.90	1.62

Table 10: Optimal design comparison for the 10-bar planar truss (Case study 8) with continuous sections.

Method	Weight: lb (kN)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Kaveh and Rahami [5]	5061.90 (22.5153)	30.67	0.1	22.87	15.34	0.1	0.46	7.48	20.96	21.70	0.1
Schmit and Farshi [11]	5089.0 (22.6359)	33.43	0.1	24.26	14.26	0.1	0.1	8.39	20.74	19.69	0.1
Schmit and Miura [12]	5076.85 (22.5818)	30.67	0.1	23.76	14.59	0.1	0.1	8.59	21.07	20.96	0.1
Schmit and Miura [12]	5107.3 (22.7173)	30.57	0.37	23.97	14.73	0.1	0.36	8.55	21.11	20.77	0.32
Venkayya [13]	5084.9 (22.6176)	30.42	0.13	23.41	14.91	0.10	0.10	8.70	21.08	21.08	0.19
Gellatly and Berke [14]	5112.0 (22.7382)	31.35	0.1	20.03	15.60	0.14	0.24	8.350	22.21	22.06	0.1
Dobbs and Nelson [15]	5080.0 (22.5958)	30.50	0.1	23.29	15.43	0.1	0.21	7.65	20.98	21.82	0.1
Rizzi [16]	5076.66 (22.5810)	30.73	0.1	23.93	14.73	0.1	0.1	8.54	20.95	21.84	0.1
Khan and Willmert [17]	5066.98 (22.5379)	30.98	0.1	24.17	14.81	0.1	0.41	7.547	21.05	20.94	0.1
Kaveh and Hassani [8]	5095.46 (22.6899)	30.86	0.1	23.55	15.01	0.1	0.22	7.63	21.65	21.32	0.1
Present work	5059.39 (22.5041)	30.5	0.1	21.99	15.70	0.1	0.5	7.55	21	22	0.1

Table 11: Design data for a twenty-five-bar space truss (Case study 9).

Design variables			
Size variables A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; q_1; q_2; q_3; q_4; q_5; q_6; q_7			
Material property and constraint data			
Young's modulus: $E = 1\text{e}7$ psi Density of the material: $\rho = 0.1\ \text{lb/in}^3 = 0.00277\ \text{kg/cm}^3$			
Stress constraints			
$\lvert\sigma_i\rvert \leq 40$ ksi (275.8 MPa); $i = 1, \ldots, 25$			
Displacement constraint in the directions of X and Y in the co-ordinate system			
$\lvert\Delta_i\rvert \leq 0.35$ in (0.8890 cm); $i = 1, 2$			
List of the avalible profiles			
Case 1: (Discrete sections) $A_i = \{0.1, 0.5 \times I(I = 1, 2, \ldots, 76), 39.81, 40\}\ \text{in}^2$ $A_i = \{0.6452, 3.2258 \times I(I = 1, 2, \ldots, 76), 256.8382, 258.0640\}\ \text{cm}^2$ Case 2: (Continuous sctions) $A_i \geq 0.1\ \text{in}^2$ (0.6452)			
Loading data			
Node	Px: kips (kN)	Py: kips (kN)	Pz: kips (kN)
1	1 (4.448)	−10 (44.48)	−10 (44.48)
2	0	−10 (44.48)	−10 (44.48)
3	0.5 (2.224)	0	0
6	0.6 (2.6688)	0	0

minimum weight is compared to that obtained by Kaveh and Rahami in [5], and Kaveh and Hassani in [8] for the same example. The result of comparison is shown in Table 7. Similar to the other cases, CSS with supervisor agents have shown a better performance. Kaveh and Rahami in [5] used a different formulation to impose the analysis criteria as a constraint. In this method, using the derivative of U^c in Eq. (6), with respect to $\{\boldsymbol{q}\}$, leads to:

$$\frac{\partial U^c}{\partial q} = \left[\boldsymbol{H_{qp}}\right]\{\boldsymbol{p}\} + \left[\boldsymbol{H_{qq}}\right]\{\boldsymbol{q}\} = \boldsymbol{0}. \tag{19}$$

Eq. (19) indicates that the complementary energy of the structure is equal to its minimum value in the compatibility condition. Thus, $\{\boldsymbol{q}\}$ should be selected such that Eq. (19) holds.

The left hand of this equation is a zero vector and it should be changed to a scalar. The best way is calculation of the norm, because the norm of a vector is equal to zero when all the entries are equal to zero. Here, we use the equilibrium itself. For this purpose we can write:

$$F(\boldsymbol{q}, \boldsymbol{A}) = W(\boldsymbol{A})(1 + \alpha\,\text{norm}([\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + [\boldsymbol{H_{qq}}]\{\boldsymbol{q}\})). \tag{20}$$

Having $\{\boldsymbol{q}\}$ and $\{\boldsymbol{A}\}$, the magnitude of F can be calculated from Eq. (20) and its minimum for a large value of α corresponds to minimum W. Other constraints, such as stress constraints, displacement constraints or dynamical properties constraints can be applied to Eq. (20) after normalizing and selecting a penalty coefficient. Therefore, the final formulation will be as follows:

$$\text{Find} \longrightarrow \boldsymbol{q}, \boldsymbol{A}; \quad A \in \left\{S_d \quad \text{or} \quad S_c\right\}$$

$$\text{Min} F(\boldsymbol{q}, \boldsymbol{A})$$

$$= \sum_{i=1}^{ne} A_i l_i \rho_i (1 + \alpha\,\text{norm}([\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + [\boldsymbol{H_{qq}}]\{\boldsymbol{q}\})) + \sum_{m=1}^{nc} \max(0, g_m(\boldsymbol{A})), \tag{21}$$

where S_d and S_c are the discrete and continuous sections, respectively. $g_m(A)$ corresponds to violation of the constraints, and ne and nc are the number of elements and number of constraints, respectively. The first summation in the right hand of Eq. (21) calculates the weight of the structure and satisfies the analysis criteria. The second summation satisfies other constraints. Because of indirect analysis, internal forces in earlier iterations are not reliable. In other words, since the redundant forces are not exact, the calculated constraints are not exact either, and cannot be relied on. Reliability criteria can be norm $\left(\left[\boldsymbol{H_{qp}}\right]\{\boldsymbol{p}\} + \left[\boldsymbol{H_{qq}}\right]\{\boldsymbol{q}\}\right)$. Accordingly, the design constraint penalty function can be altered to:

$$F(q, A) = \sum_{i=1}^{ne} A_i l_i \rho_i (1 + \alpha\,\text{norm}([\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + [\boldsymbol{H_{qq}}]\{\boldsymbol{q}\})) + \sum_{m=1}^{nc} \max(0, g_m(A))^{R(\text{norm})}, \tag{22}$$

Table 12: Optimal design comparision for the twenty-five-bar space truss (Case study 9).

Method	Weight: lb (kN)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Rajeev [10]	546.01 (2.4287)	0.1	1.8	2.3	0.2	0.1	0.8	1.8	3.0
Erbatur [18]	493.80 (2.1964)	0.1	1.2	3.2	0.1	1.1	0.9	0.4	3.4
Kaveh and Kalatjari [19]	480.23 (2.1361)	0.1	0.1	3.5	0.1	2.0	1.0	0.1	4.0
Kaveh and Rahami (C.1) [5]	479.75 (2.1340)	0.1	0.5	3.0	0.1	2.0	1.0	0.1	4.0
Kaveh and Rahami (C.2) [5]	467.6293 (2.0800)	0.1	0.1	3.7598	0.1	1.8552	0.7755	0.1408	3.8460
Present work (C.1)	479.75 (2.1340)	0.1	0.5	3.0	0.1	2.0	1.0	0.1	4.0
Present work (C.2)	467.7457 (2.080)	0.1000	0.1001	3.7449	0.1005	1.8740	0.7836	0.1425	3.8414

Table 13: Design data for a 120-bar space dome (Case study 10).

Design variables

Vaariables: A_1; A_2; A_3; A_4; A_5; A_6; A_7; q_1; q_2; q_3; q_4; q_5; q_6; q_7; q_8; q_9

Material property and constraint data

Young's modulus: $E = 304\,50$ ksi $= 210\,000$ MPa.
Density of the material: $\rho = 0.288$ lb/in^3 $= 7971.810$ kg/cm^3
For all members: $0.775 \le A_i \le 20$ in^2; $i = 1, \ldots, 120$

Constraints

$\lambda_i = \frac{l_i}{r}\ r = \sqrt{0.4 \times A}$

Stress constraints

For tensile members
$F_a \le 0.6F_y$ and $\lambda_i \le 300$
For compressive members
$\lambda_i \le 200$

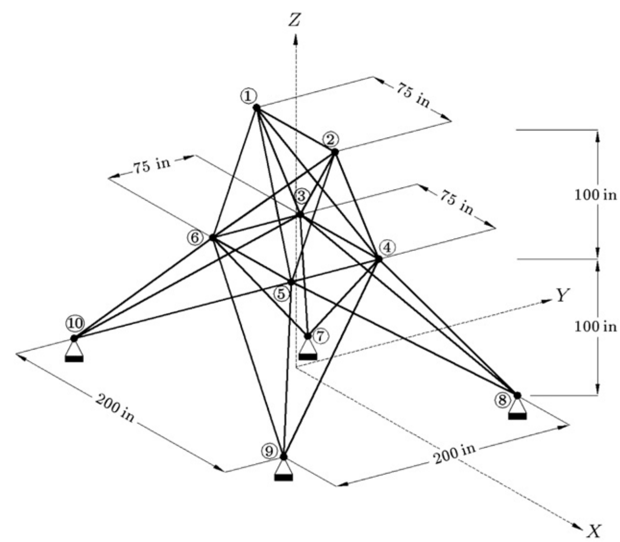
$F_a = \frac{\left[(1-\frac{\lambda_i^2}{2C_c^2})F_y\right]}{(\frac{5}{3}+\frac{3\lambda_i}{8C_c}-\frac{\lambda_i^3}{8C_c^3})}$ for $\lambda_i \le C_c$

$F_a = \frac{12\pi^2 E}{23\lambda_i^2}$ for $\lambda_i \le C_c$

$\sigma_i < 58.0$ ksi (400 MPa); $i = 1, \ldots, 120$

Displacement constraint in the directions of X, Y and Z in all unsupported nodes

$|\Delta_i| \le 0.1969$ in



Group number	Members
1	1-2
2	1-4,2-3,1-5,2-6
3	2-5,2-4,1-3,2-6
4	3-6,4-5
5	3-4,5-6
6	3-10,6-7,4-9,5-8
7	3-8,4-7,6-9,5-10
8	3-7,4-8,5-9,6-10

Figure 12: Geometry of a twenty-five-bar space truss (Case study 9) and grouping of the members.

where R(norm) is a function of norm($[\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + [\boldsymbol{H_{qq}}]\{\boldsymbol{q}\}$). This function can be considered as follows:

$$R(\text{norm}) = \log(10 + \text{NORM}), \tag{23}$$

where NORM is equal to norm($[\boldsymbol{H_{qp}}]\{\boldsymbol{p}\} + [\boldsymbol{H_{qq}}]\{\boldsymbol{q}\}$). In all examples studied in the following, Eq. (22) has been used in the CSS algorithm.

Case study 8: A ten-bar planar truss

The ten-bar truss, as shown in Figure 11, is considered for optimal design. Table 8 contains the necessary data. As seen, the displacement constraint is added to the design procedure. Two cases are considered, the first is an optimal design using discrete sections and the second corresponds to continuous sections. Eq. (22) is used as the objective function in the CSS, where a population of 20 CPs is used. In both cases, $\boldsymbol{A}$ and $\boldsymbol{q}$ are variables. In discrete cases, a code is utilized that moves the section between two available sections to one of them based on a probabilistic function. Results are obtained in Tables 9 and 10 for discrete and continuous sections, respectively.

Case study 9: A twenty-five-bar space truss

Geometry, nodal ordering and grouping of members are shown in Figure 12. Table 11 contains the necessary data for design. Table 12 contains the results and shows the efficiency of this method and combining the CSS and force method compared to the other algorithms.

In this example, the calculated maximum displacement in Case I and Case II, using the exact displacement method, are equal to 0.3482 in (0.8844 cm) and 0.3500 in (0.8890 cm), and those of the present method are 0.3496 in (0.8879 cm) and 0.3493 in (0.8871 cm), respectively. There is another set of areas for Case 2, as $A = \{0.10,\ 0.10,\ 3.7598,\ 0.10,\ 1.8932,\ 0.7755,\ 0.1408,\ 3.8460\}$, and the corresponding weight is equal to 468.1998 kN. Maximum displacement of this set of areas leads to 0.3497 in (0.8882 cm).

Case study 10: A 120-bar dome

A 120-bar dome structure is considered in this example. This structure has 9 degrees of statical indeterminacy. The necessary data for the design and the constraints are shown in Table 13. Optimal design comparison for the 120-bar dome is obtained in Table 14. Geometry, ordering and member grouping structures are shown in Figure 13. The loading condition is considered as:

1. Vertical load at node 1 equal to −13.49 kips (−60 kN).
2. Vertical loads at nodes 2 through 14 equal to −6.744 kips (−30 kN).
3. Vertical loads at the rest of the nodes equal to −2.248 kips (−10 kN).

Table 14: Optimal design comparison for the 120-bar dome (Case study 10).

Element group	Optimal cross-sectional areas (in^2)							Best weight (lb)
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	
Kaveh et al. (IACS) [20]	3.026	15.060	4.707	3.100	8.513	3.694	2.503	33320.52
Kaveh and Talatahari (PSOPC) [21]	3.040	13.149	5.646	3.143	8.759	3.758	2.502	33481.20
Kaveh and Talatahari (PSACO) [21]	3.026	15.222	4.904	3.123	8.341	3.418	2.498	33263.90
Kaveh and Talatahari (HPSACO) [21]	3.095	14.405	5.020	3.352	8.631	3.432	2.499	33248.90
Kaveh and Talatahari (HBB-BC) [22]	3.037	14.431	5.130	3.134	8.591	3.377	2.500	33287.90
Kaveh and Talatahari (CSS) [6]	3.027	14.606	5.044	3.139	8.543	3.367	2.497	33251.90
Present work	3.0129	14.7596	5.1118	3.1304	8.5430	3.2026	2.4917	33241.99

Table 15: Design data and constraints of 26-story truss (Case study 11).

Design variables
Variables $A_1; A_2; A_3; \ldots; A_{59}; q_1; q_2; \ldots; q_{246}$
Material property and constraint data
Young's modulus: $E = 1\text{e}7$ psi Density of the material: $\rho = 0.1$ lb/in^3 $= 0.00277$ kg/cm^3
Stress constraints
$\lvert\sigma_i\rvert \leq 25$ ksi(172.375 MPa); $i = 1, \ldots, 942$
Displacement constraint in the directions of X and Y in the co-ordinate system
$\lvert\Delta_i\rvert \leq 15$ in (About 1/250 of the totla height of the tower) for the four nodes of the top level in the x, y and z directions
List of the avalible profiles
Case 1: $Ai \geq 1$ in^2(6.452) cm^2 $Ai < 200$ in^2(1290.32) cm^2

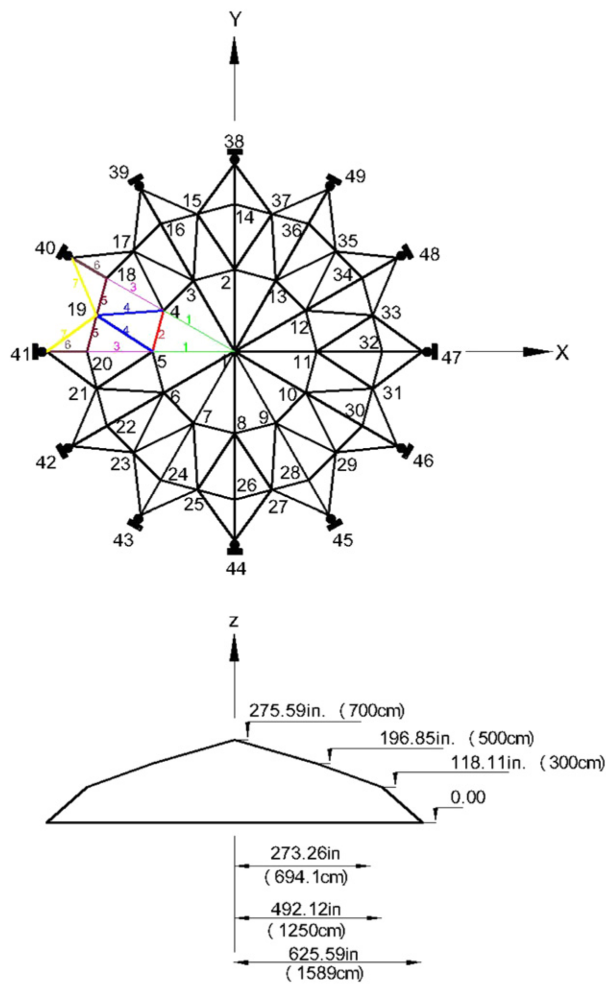


Figure 13: A 120-bar dome (Case study 10).

Redundant forces are considered as the reactions at nodes 39, 43 and 47. For the present approach, the maximum stress ratio is equal to 0.9552, the maximum displacement using the exact displacement method is equal to 0.17335 in, and the maximum displacement using the present method is calculated as 0.17339 in.

In this example, when the displacement method is utilized as an analysis procedure, the unknowns change from redundant forces to nodal displacements. Then, the number of unknowns drastically increases from 9 redundant forces to 111 nodal displacements. This imposes a highly computational cost on the optimization procedure. Eq. (24) will be used for analysis using the displacement method:

$$\text{norm}\left([\boldsymbol{K}]\{\boldsymbol{X}\} - \{\boldsymbol{F}\}\right) = 0, \qquad (24)$$

where $\boldsymbol{K}$ is considered the stiffness matrices of the structure, $\boldsymbol{X}$ is considered the nodal displacement vector and $\boldsymbol{K}$ is the nodal forces vector. Computational time for this example is obtained as 15.4568 s, achieved with a computer; Core$^{\text{TM}}$2 Duo T9550 @ 2.66 GHz 2.67 GHz processor and 4.00 GB RAM.

Case study 11: A 26-story tower

The main aim of the present method is to avoid the computation of the inverse of the large-scale structures matrices. This method must be applied to the large-scale structures to show the superiority of the present method. For this purpose, a 26-story tower, as shown in Figure 6, is considered. The loading condition is defined in Case study 4. Design data and constraints are maintained in Table 15. This structure has 246 degrees of statical indeterminacy. The member grouping has fifty-nine groups, as shown in Figure 6. The simultaneous analysis, design and optimization of this structure have 305 variables. A population of 100 CPs is considered in the CSS algorithm. Eq. (22) is taken as the objective function in the CSS algorithm with supervisor agents. Optimal design comparison for the 26-story tower is provided in Table 16.

In this example, the exact maximum nodal displacement calculated for the four top nodes, using the displacement method, is 14.3442 in. The present method leads to 14.7688 in. The maximum stress ratio is equal to 94.90%. According to Table 16, the efficiency of the CSS and, especially, the present method, in the analysis, design and optimization of large-scale structures in comparison to other methods, becomes apparent. Computational time for this example is obtained as 3040 s, achieved with a computer having a Core$^{\text{TM}}$2 Duo T9550 @ 2.66 GHz 2.67 GHz processor and 4.00 GB RAM.

Table 16: Design comparison for the 26-story truss (Case study 11).

Variable (in^2)	Erbatur and Hasancebi [23]	Rahami et al. [24]	Kaveh and Talatahari [6]	Present work
A_1	1	2.7859	0.962	1.0376
A_2	1	1.3572	2.557	2.0424
A_3	3	5.0362	1.65	1.6003
A_4	1	2.2398	0.402	1.0113
A_5	1	1.2226	0.657	1.0033
A_6	17	14.9575	18.309	2.5260
A_7	3	2.9568	0.346	1.0001
A_8	7	10.9038	3.076	1.0981
A_9	20	14.4177	2.235	2.4705
A_{10}	1	3.709	3.813	1.0222
A_{11}	8	5.7076	0.856	1.2531
A_{12}	7	4.9264	1.138	1.0024
A_{13}	19	14.1751	3.374	1.8253
A_{14}	2	1.9043	0.573	1.0463
A_{15}	5	2.8101	19.53	1.6020
A_{16}	1	1	1.512	1.0760
A_{17}	22	18.807	2.667	2.2508
A_{18}	3	2.6151	0.478	1.0177
A_{19}	9	12.5328	17.873	3.4032
A_{20}	1	1.1314	0.335	1.0012
A_{21}	34	30.5122	2.78	5.3252
A_{22}	3	3.346	0.43	1.0003
A_{23}	19	17.045	3.048	4.4083
A_{24}	27	18.0785	5.112	10.7550
A_{25}	42	39.2717	19.352	5.0916
A_{26}	1	2.6062	0.476	1.0029
A_{27}	12	9.8303	2.887	5.5097
A_{28}	16	13.1126	19.5	7.9683
A_{29}	19	13.6897	4.772	4.4314
A_{30}	14	16.9776	5.063	5.3373
A_{31}	42	37.6006	15.175	6.7094
A_{32}	4	3.0602	1.176	1.6518
A_{33}	4	5.5106	0.839	3.1108
A_{34}	4	1.8014	1.394	1.0434
A_{35}	1	1.1568	0.153	1.2485
A_{36}	1	1.2423	0.247	1.0746
A_{37}	62	62.7741	18.673	6.8163
A_{38}	3	3.3276	0.696	1.2514
A_{39}	2	4.2369	1.395	5.4658
A_{40}	4	1.7202	0.422	1.1308
A_{41}	1	1.0148	0.417	1.3079
A_{42}	2	5.6428	0.679	1.0063
A_{43}	77	78.0094	19.584	9.9490
A_{44}	3	3.2206	0.533	1.1061
A_{45}	2	3.5934	1.64	7.3345
A_{46}	3	4.7668	0.618	2.3035
A_{47}	2	1.1531	0.531	2.3722
A_{48}	3	2.1698	1.374	1.0706
A_{49}	100	99.6406	19.656	13.9159
A_{50}	4	4.1469	0.888	2.7680
A_{51}	1	2.16	4.456	5.2249
A_{52}	4	4.1499	0.386	1.0024
A_{53}	6	11.207	10.398	11.7689
A_{54}	3	11.0904	18.834	12.1676
A_{55}	49	35.94999	18.147	19.9929
A_{56}	1	2.1937	3.28	9.2241
A_{57}	62	66.1705	2.972	1.0313
A_{58}	1	3.3402	4.927	8.1362
A_{59}	3	4.0525	0.288	1.0025
Weight, lb (kN)	143436 (637.99)	142295.75 (632.92)	47370.8412 (210.70)	47108.4972 (209.536)

5. Conclusions

In this article, an efficient method is presented to avoid finding the inverse of the large matrices, especially in large-scale structures. Then, the main application of this method is in the analysis, design and optimization of large-scale structures. The CSS algorithm and force method are applied simultaneously for the analysis and design of various kinds of large-scale structures. The results obtained for large-scale examples show the accuracy of this method in handling the simultaneous analysis, design and optimization of this type of structure. A new formulation of a penalty function is presented by considering the reliability of the calculated redundant forces, and is applied to the examples. The CSS algorithm, with supervisor agents and a new formulation of the penalty function, performs well in all the examples. Also, the calculated computational time illustrates the efficiency of this method and presented formulation in the case of large-scale structures. As

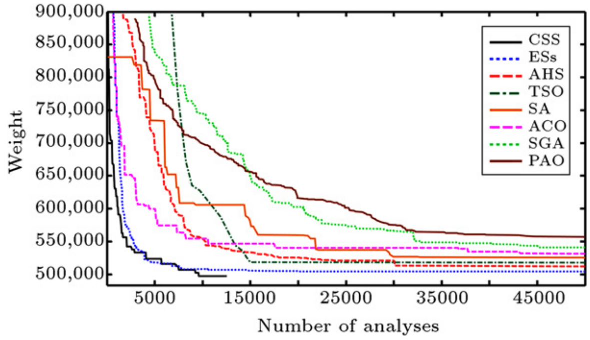


Figure 14: Comparison of the convergence history of the various optimization algorithms [25].

shown in Figure 14, the CSS algorithm has a good performance in comparison to other optimization techniques [25]. The results of these examples illustrate the capability of the CSS algorithm and force method when simultaneously utilized for analysis, design and optimization of structures.

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